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we may say that to distinguish between consciousness and object, or to recognize that an object existed prior to our experience of it, is not to assume a comparison between the experienced and the unexperienced, but is to deal with the "meaning" of things, a meaning which must be construed in terms of the fringe on the one hand and of bodily control on the other.

As was intimated previously, the significance of this treatment of consciousness lies in the interpretation which it gives to experience. The pragmatic movement of our day is, above all, an attempt to reinterpret philosophic problems in the light of this conception. It holds out the hope that many difficulties which so far have resisted explanation will be overcome when approached from this new standpoint.

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## INTERESTING THEOREMS IN SYMBOLIC LOGIC

TWO propositions in the algebra of implication or "calculus of propositions" have been much discussed. They are: "A false proposition implies any proposition," and "A true proposition is implied by any proposition." These theorems have been hailed as discoveries and repudiated as absurdities. But on all sides the impression seems to prevail that these two are *sui generis* in the algebra. For this reason, it may be worth while to present a partial list of propositions which are of the same kind, involve the same principles, and can be proved from the same assumptions. Comparatively few such theorems have been printed, but their number is apparently infinite. Reference, following the proposition, indicates those given in "Principia Mathematica."[1]

1. $q \supset (p \supset q)$ ($*2{\cdot}02$). A true proposition is implied by any proposition.

2. $\sim p \supset (p \supset q)$ ($*2{\cdot}21$). A false proposition implies any proposition.

3. $p \supset (\sim p \supset q)$ ($*2{\cdot}24$). If $p$ is true, then not-$p$ implies any proposition.

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[1] The notation is that of "Principia Mathematica" with some modifications. $p$, $q$, $r$ represent propositions. $\supset$ symbolizes "implies." $\sim$, placed before a letter or expression, is the sign of negation. $\sim p$ may be read "not-$p$" or "$p$ is false." Similarly $p$ may be read as written or as "$p$ is true." The product $pq$ means "$p$ and $q$ are both true"; $\sim p \sim q$, "$p$ and $q$ are both false," etc. $+$ is the sign of disjunction. $p + q$ is read "either $p$ is true or $q$ is true." For the sake of clearness, I render the main implication sign in each theorem by "if . . . then," subordinate implications by "implies."

4. $\sim (p \supset q) \supset (\sim p \supset q)$ ( 2·5). If $p$ does not imply $q$, then not-$p$ implies $q$.

5. $\sim (p \supset q) \supset (p \supset \sim q)$ (*2·51). If $p$ does not imply $q$, then $p$ implies not-$q$.

6. $\sim (p \supset q) \supset (\sim p \supset \sim q)$ (*2·52; misprinted, note proof). If $p$ does not imply $q$, then not-$p$ implies not-$q$.

7. $\sim (p \supset q) \supset (q \supset p)$ (*2·521). If $p$ does not imply $q$, then $q$ implies $p$.

8. $\sim (p \supset q) \supset p$. If $p$ does not imply $q$, then $p$ is true.

9. $\sim (p \supset q) \supset \sim q$. If $p$ does not imply $q$, then $q$ is false. 8 and 9 are even more startling than 1 and 2, from which they are proved by the rule $(p \supset q) \supset (\sim q \supset \sim p)$.

10. $\sim (p \supset q) \supset p \sim q$. If $p$ does not imply $q$, then $p$ is true and $q$ false.

11. $pq \supset (p \supset q)$. (*3·4). If $p$ and $q$ are both true, then $p$ implies $q$.

12. $pq \supset (q \supset p)$. If $p$ and $q$ are both true, then $q$ implies $p$.

13. $pq \supset (\sim p \supset q)$. If $p$ and $q$ are both true, then not-$p$ implies $q$.

14. $pq \supset (\sim q \supset p)$. If $p$ and $q$ are both true, then not-$q$ implies $p$.

15. $\sim p \sim q \supset (p \supset q)$. If $p$ and $q$ are both false, then $p$ implies $q$.

16. $\sim p \sim q \supset (q \supset p)$. If $p$ and $q$ are both false, then $q$ implies $p$.

17. $\sim p \sim q \supset (p \supset \sim q)$. If $p$ and $q$ are both false, then $p$ implies not-$q$.

18. $\sim p \sim q \supset (q \supset \sim p)$. If $p$ and $q$ are both false, then $q$ implies not-$p$.

19. $\sim pq \supset (p \supset q)$. If $p$ is false and $q$ true, then $p$ implies $q$.

20. $\sim pq \supset (p \supset \sim q)$. If $p$ is false and $q$ true, then $p$ implies not-$q$.

21. $\sim pq \supset (\sim q \supset \sim p)$. If $p$ is false and $q$ true, then not-$q$ implies not-$p$.

22. $\sim (p + q) \supset (p \supset q)$. If it is false that "either $p$ is true or $q$ is true," then $p$ implies $q$.

In 22 the $\sim p \sim q$ of 15 is replaced by its equivalent, $\sim (p + q)$. Three more such theorems result from a similar substitution in 16, 17, and 18.

23. $\sim (\sim p + \sim q) \supset (p \supset q)$. If it is false that "either $p$ is false or $q$ is false," then $p$ implies $q$.

23 results from substituting $\sim (\sim p + \sim q)$ for $pq$ in 11. Three more theorems result from a similar substitution in 12, 13, and 14.

24. $\sim (\sim p + q) \supset (\sim p \supset q)$. If it is false that "either $p$ is false or $q$ is true," then not-$p$ implies $q$.

24 results from substituting $(\sim p + q)$ for its equivalent, $(p \supset q)$,

in 4. Three more theorems result from a similar substitution in 5, 6, and 7.

The above are the simplest of the theorems of this character which involve only the terms, $p$, $q$, $\sim p$ and $\sim q$. Without increasing the number of terms, more complex theorems of the same sort can be proved, apparently without limit. A few of the simpler theorems involving a third proposition sign, $r$, are as follows:

25. $\sim (p \supset q) \supset (r \supset p)$. If $p$ does not imply $q$, any proposition $r$ implies $p$.

26. $\sim (p \supset q) \supset (q \supset r)$. If $p$ does not imply $q$, $q$ implies any proposition $r$.

27. $\sim (p \supset q) \supset (\sim p \supset r)$. If $p$ does not imply $q$, not-$p$ implies any proposition $r$.

28. $\sim (p \supset q) \supset (r \supset \sim q)$. If $p$ does not imply $q$, then any proposition $r$ implies not-$q$.

29. $(p \supset q) \supset [p \supset (r \supset q)]$. If $p$ implies $q$, then $p$ implies that any proposition $r$ implies $q$.

30. $(p \supset q) \supset [p \supset (\sim q \supset r)]$. If $p$ implies $q$, then $p$ implies that not-$q$ implies any proposition $r$.

31. $(p \supset q) \supset [\sim q \supset (r \supset \sim p)]$. If $p$ implies $q$, then not-$q$ implies that any proposition $r$ implies not-$p$.

32. $(p \supset q) \supset [\sim q \supset (p \supset r)]$. If $p$ implies $q$, then not-$q$ implies that $p$ implies any proposition $r$.

33. $\sim (p \supset q) \supset [r \supset (q \supset p)]$. If $p$ does not imply $q$, then any proposition $r$ implies that $q$ implies $p$.

34. $\sim (p \supset q) \supset [r \supset (\sim q \supset p)]$. If $p$ does not imply $q$, then any proposition $r$ implies that not-$q$ implies $p$.

35. $\sim (p \supset q) \supset [r \supset (p \supset \sim q)]$. If $p$ does not imply $q$, then any proposition $r$ implies that $p$ implies not-$q$.

The addition of a fourth proposition sign constitutes a vocabulary sufficient for Alice in Wonderland. But consideration for the reader—and the printer—requires that we make an end.

Any one of these theorems can be proved from the postulates of "Principia Mathematica," from those of Peano, from Schröder's, and from any of the sets given by Huntington. They can also be proved, in somewhat different form, from the assumptions of Mrs. Ladd-Franklin's algebra, if the variables of that system are taken to symbolize propositions or propositional functions.

What these theorems reveal is the divergence of the meaning of "implies" in the algebra of logic from the "implies" of valid inference. The way in which such theorems came to be included in the "calculus of propositions" is, briefly, this: The calculus of propositions was preceded by and grew out of the "calculus of classes." In the algebra of classes, "all *a* is *b*" may be interpreted, "all cases of *a* are cases of *b*,'' or ''all cases in which *p* is true are cases in which *q* is true,''—hence, finally ''*p* implies *q*.'' But the implication relation—when ''*p* implies *q*'' means ''*q* can be inferred from *p*''—fails to be analogous to the relation of class inclusion in more ways than have yet been noticed. Many of the above theorems can be interpreted as propositions in the algebra of classes, and when so construed become intelligible and true. Most of them will then be disclosed as results of the two conventions, (1) the zero or empty class is contained in every class, and (2) every class is contained in the universe of discourse. These are the exact analogues of (1) a false proposition implies any proposition, and (2) a true proposition is implied by any proposition. In order successfully to symbolize valid inference, the algebra of implication needs radical revision.

The consequences of this difference between the ''implies'' of the algebra and the ''implies'' of valid inference are most serious. Not only does the calculus of implication contain false theorems, but all its theorems are *not proved.* For the theorems are implied by the postulates in the sense of ''implies'' which the system uses. The postulates have not been shown to imply any of the theorems except in this arbitrary sense. Hence, *it has not been demonstrated* that the theorems *can be inferred from* the postulates, even if all the postulates are granted. The assumptions, *e. g.*, of ''Principia Mathematica,'' imply the theorems in the same sense that a false proposition implies anything, or the first half of any of the above theorems implies the last half. The postulates of the ''Principia'' imply the ''consequences'' thereafter set down in exactly the same fashion that ''Socrates was a solar myth'' implies ''All triangles have two or more sides.''

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## REVIEWS AND ABSTRACTS OF LITERATURE

*The Meaning of God in Human Experience: A Philosophic Study of Religion.* William Ernest Hocking. New Haven: Yale University Press. 1912.

This is an interesting and important book. It is interesting because of its subject, for it deals with the central theme both of philosophy and religion, the meaning of God in human experience. It is interesting because of its method, for it is at all points the work of a fresh, original thinker, dealing at first hand with his subject, and bringing to each question which he faces at once a wide knowledge and an open mind. It is interesting, finally, because of its conclusion, for the line which the author's thought follows leads him away from the beaten track of contem-